Stability of the Equilibrium Position of the Centre of Mass of an Inextensible Cable - Connected Satellites System in Circular Orbit

Vijay Kumar[1], Nikky Kumari[2]
*(Assistant Professor, Department of Mathematics, Shobhit University, Gangoh, Saharanpur,India)
** (Research Scholar, Department of Mathematics, Dayalbagh Educational Institute, Agra, India)

ABSTRACT: *In this paper we have studied the motion and stability of the centre of mass of a system of two satellites connected by inextensible cable under the influence of air resistance and magnetic force in the central gravitational field of oblate earth in circular orbit. We have obtained an equilibrium point which has been shown to be stable in the sense of Liapunov.*

Keywords: *Perturbative forces, stability, interconnected satellites, Equilibrium point, and Circular orbit.*

I. INTRODUCTION

This paper is devoted to study the equilibrium position under the influence of air resistance and magnetic force of oblate earth in case of circular orbit of the centre of mass of the system.

For this, firstly we have derived equations of motion in case of circular orbit of the centre of mass of the system under perturbative forces mentioned above and then Jacobian integral for the problem is obtained. Equilibrium Position has been obtained shown to be stable in the sense of Liapunov. This work is direct generalization of works done by V.V. Beletsky; R. B. Singh; B. Sharma; S. K. Das; P. K. Bhattacharyya and C.P.Singh.

II. EQUATIONS OF MOTION OF ONE OF THE TWO SATELLITES IN ELLIPTIC ORBIT

The equations of motion of one of the two satellites when the centre of mass moves along a keplerian elliptical orbit in Nechvill's co-ordinate system have been derived in the form.

$$x''-2y'-3\rho x=\bar{\lambda}_{\propto}\rho^4 x-\frac{4Ax}{\rho}-\frac{B\cos i}{\rho}-f\rho\rho'$$

$$y''+2x'=\bar{\lambda}_{\propto}\rho^4 y-\frac{A}{\rho}y-\frac{B\rho'\cos i}{\rho^2}-f\rho^2$$

$$z''+z=\bar{\lambda}_{\propto}\rho^4 z-\frac{A}{\rho}z-\frac{B}{\rho}\left[\frac{\rho'}{P}\cos(v+w)+\frac{1}{\mu_E}\left(3p^3\rho^2-\mu E\right)\sin(v+w)\right]\sin i \qquad \text{……....... (2.1)}$$

Where,

$\bar{\lambda}_{\propto}=\frac{p^3}{\mu}\lambda_{\propto}=\frac{p^3}{\mu}\frac{\lambda(m_1+m_2)}{l_0 m_1 m_2}$: λ being Lagrange's multiplier's and m_1, m_2 being masses of two satellites.

l_0 being the length of cable connected by two satellites

$\rho=\frac{R}{p}=\frac{1}{1+e\cos v}$; p being focal parameter and e eccentricity of the orbit of centre of mass

R = Radius vector of the centre of mass from the attracting centre
v = True anomaly of the centre of mass
i = Inclination of the orbit of centre of mass with the equatorial plane of the earth

$A=\frac{-k_2}{p^2}$ = oblateness force parameter

B = $\frac{m_1}{m_1+m_2}\left[\frac{Q_1}{m_1}\frac{-Q_1}{m_2}\right]\frac{\mu_E}{\sqrt{\mu p}}$ = magnetic force parameter

$f=\frac{a_1 p^3}{\sqrt{\mu p}}$ = Air resistance force parameter

Here, dashes denote differentiations w.r.t. to true anomaly v.

The condition of constraint is given by

$$x^2+y^2+z^2\le\frac{1}{\rho^2} \qquad \text{.................. (2.2)}$$

Since the general solution of the system of differential equations given by (2.1) is beyond our reach so we restrict ourselves to the case of circular orbit of the centre of mass of the system in equatorial plane (i = 0) and hence we get from (2.1) on putting.

$\rho = \dfrac{1}{1+e\cos v} = 1$, $\rho' = 0$ and $i = 0$ (for equatorial plane)

$$x'' - 2y' - 3x = \bar{\lambda}_\alpha x - 4Ax - B$$

$$y'' + 2x' = \bar{\lambda}_\alpha y + Ay - f$$

$$z'' + z = \bar{\lambda}_\alpha z + Az \qquad \text{.................... (2.3)}$$

The condition of constraint given by (2.2) takes the form

$$x^2 + y^2 + z^2 \le 1 \qquad \text{.................. (2.4)}$$

Thus if inequality sign holds in (2.4) then the motion takes place with loose string and the motion is called free motion. If the equality sign holds in (2.4), then the motion takes place with tight string and the motion is called constrained motion.

We are actually interested in stability due to constrained motion. Thus, motion takes place on unit sphere given by -

$$x^2 + y^2 + z^2 = 1 \qquad \text{.................. .(2.5)}$$

Differentiating (2.5), we get

$$xx^1 + yy^1 + zz^1 = 0 \qquad \text{.................. (2.6)}$$

To obtain Jacobian integral of the problem, we multiply the first, second and third equation of (2.3) by x', y' and z' respectively and add them together, we get after integrating on using (2.5) and (2.6)

$$x'^2 + y'^2 + z'^2 - \left(3x^2 - z^2\right) = 5Ax^2 - 2Bx - 2fy + h \qquad \text{.................. (2.7)}$$

Which is known as Jacobian integral and can be interpreted as energy equation with modified potential given by

$$V = -\frac{1}{2}\left(3x^2 - z^2\right) - \frac{5A}{2}x^2 + Bx + fy \qquad \text{.................. (2.8)}$$

Differentiating (6) with respect to v, we get

$$x'^2 + y'^2 + z'^2 = -\left(xx'' + yy'' + zz''\right) \qquad \text{.................. (2.9)}$$

Multiplying the first, second and the third equations of (2.3) by x, y and z respectively and adding we get on using (2.5)

$$xx'' + yy'' + zz'' = 2\left(xy' - x'y\right) + \left(3x^2 - z^2\right) - \left(5x^2 - 1\right)A + \bar{\lambda}_\propto - Bx - fy \qquad \text{................ (2.10)}$$

Using (2.9) in (2.10), we get

$$-\bar{\lambda}_\propto = \left(x'^2 + y'^2 + z'^2\right) + 2\left(xy' - x'y\right) + \left(5x^2 - 1\right) + \left(3x^2 - z^2\right) - Bx - fy\,. \qquad \text{................. (2.11)}$$

To simplify (2.7) and (2.8), we use spherical polar coordinate on unit sphere:

$$x = \cos\phi\cos\psi : y = \cos\phi\sin\psi \text{ and } z = \sin\phi \qquad \text{.................. (2.12)}$$

Using (2.12), (2.7) and (2.8) become respectively

$$\phi'^2 + \psi'^2\cos^2\phi = \left(3\cos^2\psi + 1\right)\cos^2\phi - 5A\cos^2\phi\cos^2\psi - 2B\cos\phi\cos\psi - 2f\cos\phi\sin\psi + h_1$$

$$\text{.................. (2.13)}$$

Where,

$h_1 = h - 1$, and

$$V\left(\phi,\psi\right) = -\left(3\cos^2\psi + 1\right)\cos^2\phi + 5A\cos^2\phi\cos^2\psi + 2B\cos\phi\cos\psi + 2f\cos\phi\sin\psi \qquad \text{............... (2.14)}$$

III. PARTICULAR SOLUTION AND STABILITY

For equilibrium positions, (2.14) can be taken as modified potential energy.

The equilibrium positions are given by the stationary values of $v\left(\phi,\psi\right)$ and hence, we have

$$\frac{\partial V}{\partial \phi} = 0 \qquad \text{.................. (3.1)}$$

$$\frac{\partial V}{\partial \psi} = 0 \qquad \text{.................. (3.2)}$$

Differentiating (2.14) partially with respect to ϕ and using (3.1), we get

Sin ϕ = 0 i.e. ϕ = 0 ....(3.3)

Also, we have

$$\left[\frac{\partial V}{\partial \psi}\right]_{\psi=\psi_0}^{\phi=0} = 3\cos\psi_0 \sin\psi_0 - 5A\cos\psi_0 \sin\psi_0 - B\sin\psi + f\cos\psi = 0 \qquad \text{.................. (3.4)}$$

For smallest value of ψ_0, we have from (3.4) on putting $\cos\psi_0 = 1$ and $\sin\psi_0 = \psi_0$

$$\psi_0 = \frac{-f}{3-5A-B} \qquad \text{.................. (3.5)}$$

Thus, the equilibrium point is given by

$$\phi = \phi_0 = 0 \qquad \text{and} \qquad \psi = \psi_0 = \frac{-f}{3-5A-B} \qquad \text{.................. (3.6)}$$

To test the stability to the equilibrium position given by (2.6), we have

$$\left[\frac{\partial^2 V}{\partial \phi^2}\right]_{\psi=\psi_0}^{\phi=0} = 2\left(3\cos^2\psi_0 + 1\right) - 10A\cos^2\psi - 2B\cos\psi_0 - 2f\sin\psi_0 \qquad \text{.................. (3.7)}$$

$$\left[\frac{\partial^2 V}{\partial \psi^2}\right]_{\psi=\psi_0}^{\phi=0} = 6\cos 2\psi_0 - 10A\cos 2\psi_0 - 2B\cos\psi_0 - 2f\sin\psi_0 . \qquad \text{.................. (3.8)}$$

and

$$\left[\frac{\partial^2 V}{\partial \psi \partial \phi}\right]_{\psi=\psi_0}^{\phi=0} = 0 = \left[\frac{\partial^2 V}{\partial \phi \partial \psi}\right]_{\psi=\psi_0}^{\phi=0} \qquad \text{.................. (3.9)}$$

For equilibrium point given by (3.6) to be stable in the sense of Liapunov, we have to show that

$$\begin{vmatrix} \left[\frac{\partial^2 V}{\partial \psi^2}\right]_{\psi=\psi_0}^{\phi=0} & \left[\frac{\partial^2 V}{\partial \phi \partial \psi}\right]_{\psi=\psi_0}^{\phi=0} \\ \left[\frac{\partial^2 V}{\partial \phi \partial \psi}\right]_{\psi=\psi_0}^{\phi=0} & \left[\frac{\partial^2 V}{\partial \phi^2}\right]_{\psi=\psi_0}^{\phi=0} \end{vmatrix} > 0 \qquad \text{.................. (3.10)}$$

Using (3.7), (3.8) and (3.9) in (3.10) it can be easily seen that (3.10) is positive if 5A + B < 3. .

Conclusion: we conclude that the equilibrium point

$$\phi = \phi_0 = 0 ; \qquad \psi = \psi_0 = \frac{-f}{3-5A-B}$$

is stable in the sense of Liapunov if 5A + B < 3.

REFERENCE

[1] V. V. Beletsky, About the relative motion of two connected bodies in orbit, Kosmicheskiye Isseldovania,vol.7, No. 6 (1969),827-840(Russian)

[2] R. B. Singh, The three dimensional motion of two connected bodies in an elliptical orbit, Bulletin of Moscow state university, Mathematic-Mechanics. No.4, 59-64, 1973(Russian)

[3] B. Sharma, The motion of a system of two cable - connected satellites in the atmosphere, Ph.D. Thesis submitted to B.U. Muzaffarpur. (1974).

[4] S. K. Das, and P. K .Bhattacharya, Effect of magnetic force on the motion of a system of two cable connected satellites in orbit, Proc. Nat. Acad. Sci. India. (1976), 287-299

[5] C. P Singh, Motion and stability of inter-connected satellites system in the gravitational field of oblate earth. Ph.D. Thesis submitted to B.U. Muzaffarpur (1983)

[6] V. Kumar and N. Kumari, Stability of Equilibrium point of the centre of mass of an extensible cable connected satellites system in case of circular orbit in three dimensional, IJSER,Vol-4,Issue 9, (2013),1802-1808